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OPTIMISING THE DRYING OF A POROUS BODY:A SIMPLE STEADY-STATE DIFFERENTIAL APPROACH (SSDA)

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ABSTRACT

This paper sought to develop from theoretical principles an expression that ultimately illustrated a relationship between the distance or gap between the hot body and the cross sectional area of the porous body that will optimize the drying operation. This study, undertaken via an exploration of literature was able to establish the significance of drying as a unit operation as well as the general mechanism of hot air drying. Indeed, a synopsis of the dynamics of mass and heat transfer was explored facilitating the establishment of relationships relating to both mass and heat transfer while simultaneously reinforcing the view that the drying operation involves both heat and mass transfer processes. This investigation subsequently invoked the utilization of simple differential equations to approximate suggested relationships.

Indeed the study was able to demonstrate via an almost 'stage wise' approach a simple model to optimize the drying operation while also presenting a theoretical approach that established that optimizing the drying operation will involve the maintenance of a proportional relationship between the surface area of the porous body and the square of the 'gap'.

Keywords-Optimize, Mass and Heat transfer, Porous body

I. INTRODUCTION

According to Mujumdar (2006) drying refers to the process of thermally removing volatile substances (moisture) which can be present in loose chemical combination or trapped in the micro-structure of the solid and achieved via heat transfer from the environment which causes evaporation of surface moisture as well as the movement of internal moisture to the surface for evaporation. Indeed in other to achieve drying there must be a source of heat energy to convert the liquid water into water vapor, a sink, or destination, to receive the water vapor, and as well as the wet solid requiring drying (FAO 2014). While the significance of drying in industrial processes is largely undisputed with the drying of porous materials very common in the food and chemical industries as well as in agriculture, most industrial drying processes are sadly significantly energy intensive(Kelbaliev and Manafov 2009). Indeed a recognition of this unfortunate reality has in recent times prompted the need to identify possibilities for the reduction of energy consumption such as the utilization of heat ex-changers to facilitate the use of exhaust hot air which is usually discharged into the atmosphere (Rajka and Alka 2011). Thus this paper will therefore attempt to generate a simplified model that will incorporate the surface area of the porous body(A) as well as the gap between the porous body and the heat source (δ) via the application of the universal laws of mass and energy conservation. Indeed the goal of the attempted optimization process will be the determination of a relationship between selected the parameter that will result in a maximum (or minimum) of a function in this case the drying operation (Russenschuck 1999).

II. MECHANISM OF HOT AIR DRYING

According to Greensmith (1998) which it is important to recognise the factors that control rate of drying such as air temperature, humidity hot air velocity, the general mechanism of hot air drying presents a scenario that facilitates a better understanding of the process to enable modelling. He suggests that hot air drying can be summarized in three major stages :

Hot air is blown over a wet solid causing the water vapor to diffuse through a boundary film of air surrounding the solid and mass transfer is effected by the moving air. A water vapor pressure gradient is subsequently established from the moist interior of solid to the dry air, with the resultant water vapor pressure gradient therefore provides the 'driving force ' that will facilitate drying. Thus a system is presented that illustrates both mass and heat transfer.

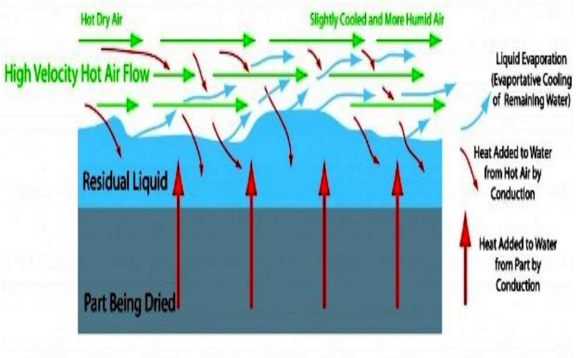

Figure 1: Mechanism of drying (Fuchs 2013)

Ryozo (1996) further suggests that with respect to this simultaneous transport of mass (water and water vapor)-heat-momentum during drying a series of 'drying equations ' can be generated and utilized as simultaneous differential equations, where mass transfer and heat transfer are aptly represented by the Fick's law of diffusion and heat equations respectively. While recognizing that heat transfer can occur via radiation and convection, with negligible conduction, Jankowsky (1995) and Rosen (1983) highlight the predominance of drying via convection with suggestions that the drying process is characterized by three distinct phases, with each phase determined by the drying rate variation , resulting in a peculiar drying characteristic curve for the porous body. Figure 2 therefore illustrates the typical drying curve for a porous body.

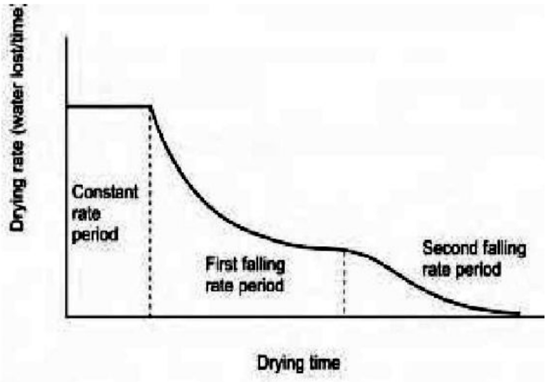

Figure 2: Characteristic drying curve for a porous material (Shaheen et al 2012)

As shown in Figure 2 the first drying period is characterised by a constant drying rate per unit of surface area. This period, according to Erle (2000) the period is characterised by a surface which is kept wet by the constant capillary-driven flow of water from within the particle. With the overall rate of drying determined by the properties of the air such as temperature, relative humidity as well as air velocity.

The falling rate period identified occur after the removal of the surface moisture or during the removal of moisture from within the porous body and is dependent on characteristics, surface temperature and the vapour pressure of the hot air (Jankowsky and Gilson 2005).

III. MODEL DEVELOPMENT

Holecek and Kohout [no Date] therefore states that the overall drying mechanism suggests both mass and heat transport proceed together during drying the limiting step of the rate of drying being the is the mass transfer. Indeed the drying rate can be expressed as an intensity of the mass flow and defined as

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$$\frac{\partial}{\partial t}\left(\frac{\partial m}{\partial A}\right) \longrightarrow$$

Where m= mass of moisture in the solid A= Surface area exposed

Assuming the model is for the control volume as illustrated in Figure 3

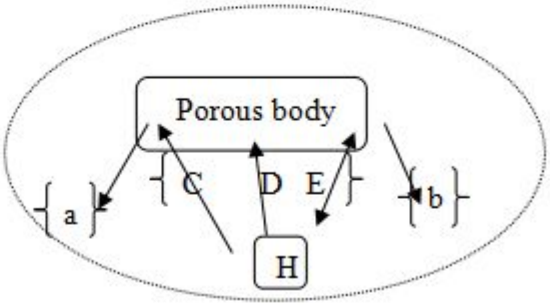

Figure 3: Simplified illustration of control volume

a: heat lost by the vaporising water
b: heat losses from the process during transfer = 0
C: Heat transfer by convection
D: Heat transfer by conduction = 0
E: Heat transfer by radiation = in both directions
H: Hot body

The Hot air drying operation will be explored as a static optimisation problem such that the relationships are developed based on the following assumptions for simplicity:

- Negligible heat transfer by conduction between the heat source and the porous material
- Steady state operational process , such that there is no heat accumulation within the system
- Heat losses are negligible
- Mass flow rate m is constant
- The average heat transfer coefficient of the porous body is constant

Since this investigation assumes heat transfer via conduction is negligible due to the poor thermal conductivity of air (Avison 1989), heat transfer will be explored via radiation and convection.

As suggested earlier this model will attempt to incorporate easily controllable variables such that the Rate of the Drying operation (D)= f(δ, A) and the temperature of the hot body source = T_h

where U= Average heat transfer coefficient of the porous body

Assuming steady state conditions for simplicity $\frac{\partial}{\partial t}(\frac{\partial m}{\partial A})$

This further simplifies as:

$^{m}/_{At}$

Thus, an inverse relationship exists between rate of drying and the surface area of the porous body.

It also satisfies logic to suggest that the greater the distance from the heat source or gap (δ) the longer the duration of drying (or the smaller the drying rate value) since the possibility of energy loss increases with distance

This analysis therefore suggests that the following relationship holds:

D = $G\,{}^{1}/_{\delta A}$ where D [${}^{Kg}/_{m^2 s}$] , with the G factor incorporating the overall heat transfer coefficient

given that $\frac{\partial D}{\partial T_h}$ = 0 will maximize the Drying operational rate , therefore differentiating the simple model with respect to the temperature of the hot body;

Recalling that U is a constant

$$\frac{\partial D}{\partial T_h} = -\left[{}^{G}/_{\delta^2 A^2} \times \left(\frac{A\partial\delta}{\partial T_h} + \frac{\delta\partial A}{\partial T_h}\right)\right] \longrightarrow$$

Major Equation 1: Basic model that will be utilised for analysis

IV. INVESTING HEAT TRANSFER VIA RADIATION

This investigation will incorporate certain assumptions for simplicity:

- The emissive and reflective properties of the porous body are constant
- The porous body both gain and losses energy via radiation

In general for the hot body, with an emissive power of E_h, so the energy leaving body will be $E_h\ A_h$. The energy leaving the hot body and absorbed by the porous is be $E_h\ A_h F_{1-2}$. Similarly the energy leaving the porous body and being absorbed by the hot body is $E_p\ A_p F_{2-1}$. The net energy interchange from the hot body to the porous body will therefore be

$$E_h\ A_h F_{1-2} - E_p\ A_p F_{2-1} = {}^{\circ}Q_{h-p}$$

Where F_{2-1} = fraction of energy leaving the porous body which reaches the hot body

F_{1-2} =fraction of energy leaving the hot body which reaches the porous body And are functions of geometry only.

however in terms of temperature according to Stefan-Boltzmann law (Wray 1975)

Whereα = Stefan-Boltzmann constant = 5.67x10^{-8}$^{\mathrm{watt}}/_{m^2K^4}$

Where for non black bodies $^{\circ}Q_{h-p}$ = $\alpha\, E_h T^4{}_h$ - $\alpha\, E_p T^4{}_p$

Heat absorbed by the moisture in the porous body

In determining this parameter two heat considerations must be considered

Heat required to raise the moisture to boiling point ($^{\circ}m_w C_{pw}(T_b - T_p)$) and the Latent heat of vaporisation ($^{\circ}m_w L$) (Mujumdar and Devahastin 2000)

Thus net heat absorbed by the moisture:

$^{\circ}Q_{mp} = {}^{\circ}m_w L + {}^{\circ}m_w C_{pw}(T_b - T_p)$ where T_b and T_p are temperatures of the boiling water and the porous body respectively.

C_{pw}= Specific Heat capacity of water

V. INVESTIGATING HEAT TRANSFER VIA CONVECTION

Heat transferred to the moisture film on the surface of the porous body:

$^{\circ}Q_{con}$ =Ah_p ($T_b - T_p$), where h_p is the film heat transfer coefficient

Heat lost by the porous body due to vaporised moisture: $^{\circ}m_w C_{pw} T_b$

Net heat absorbed by the porous body: $^{\circ}m_p C_{pp}(\Delta T_p)$

C_{pp}= Specific Heat capacity of the porous body

Therefore in developing the relationship between the surface area of the porous body (A) and the Temperature of the hot body T_h the energy equation as presented by Urieli (2014) will be explored. Therefore given that:

$\Delta {}^{\circ}Q_{net}$- $\Delta {}^{\circ}W_{net} = \Delta {}^{\circ}U$, valid for a closed system

Thus energy is transferred between the system and the surroundings in the form of heat and work, resulting in a change of internal energy of the system. The system under investigation may be considered as sufficiently represented by the energy equation:

$^{\circ}Q_{in}$- $^{\circ}Q_{out} = {}^{\circ}Q_{consumed} + {}^{\circ}Q_{accumulated}$, with $^{\circ}Q_{accumulated} = 0\ for\ steady\ state\ operations$

Where $^{\circ}Q_{in}$= heat input via convection and radiation

$^{\circ}Q_{out}$= Heat radiated back by the porous body + Heat lost due to the vapourised moisture

$^{\circ}Q_{in}$- $^{\circ}Q_{out}$= Ah_p ($T_b - T_p$) + $\alpha\, E_h T^4{}_h$ - $\alpha\, E_p T^4{}_p$ - $^{\circ}m_w C_{pw} T_b$

$^{\circ}Q_{consumed}\ in\ the\ control\ volume$= Net heat absorbed by the porous body+ net heat absorbed by the moisture in the porous body:

$={}^{\circ}m_p C_{pp}(\Delta T_p) + {}^{\circ}m_w L + {}^{\circ}m_w C_{pw}(T_b - T_p)$

Resulting expression

$$Ah_p\,(T_b - T_p) + \alpha\,E_h T^4{}_h - \alpha\,E_p T^4{}_p - {}^{\circ}m_w C_{pw} T_b = {}^{\circ}m_p C_{pp}(\Delta T_p) + {}^{\circ}m_w L + {}^{\circ}m_w C_{pw}(T_b - T_p)$$

For simplicity let where $\Delta T_{pb} = (T_b - T_p)$

$$Ah_p\ \Delta T_{pb} = {}^{\circ}m_p C_{pp}(\Delta T_p) + {}^{\circ}m_w L + {}^{\circ}m_w C_{pw}(\Delta T_{pb}) - \alpha\,E_h T^4{}_h + \alpha\,E_p T^4{}_p + {}^{\circ}m_w C_{pw} T_b$$

Differentiating with respect to T_h,

$$\frac{h_p\ \Delta T_{pb}\ \partial A}{\partial T_h} = \frac{{}^{\circ}m_p C_{pp}\ \partial \Delta T_p}{\partial T_h} - 4\alpha\ E_h T^3{}_h + 4\alpha\ E_p T^3{}_p$$

However assumed the final temperature attainable by the porous body is fixed such that

$$\frac{\partial \Delta T_p}{\partial T_h} = 0$$

$$\frac{\partial A}{\partial T_h} = 4\alpha\left[E_p T^3{}_p - E_h T^3{}_h\right] \longrightarrow$$ Major Equation 2:

Relationship between the gap(δ) and the temperature of the hot body

Assuming ficks law correctly predicts the mass flow rate of moisture from the porous body, such that for

${}^{\circ}m_w = -A\,{}^{\circ}D_w \frac{\partial Z_w}{\partial \delta}$ where

${}^{\circ}D_w$ = Diffusion Coefficient of moisture

Z_w = Concentration of the moisture in the air

${}^{\circ}m_w$ =mass Flow rate of moisture

$${}^{\circ}m_w\,\delta = A\,{}^{\circ}D_w Z_w$$

$$\frac{\partial \delta}{\partial T_h} = \frac{\partial Z_w}{\partial T_h} * \frac{A\,{}^{\circ}D_w}{{}^{\circ}m_w}$$

However since $\frac{{}^{\circ}Z_w}{{}^{\circ}Z_{ws}}$ = Relative Humidity (RH) where ${}^{\circ}Z_{ws}$ concentration of moisture that saturates the air at a given temperature (Tang 2008)

$$\frac{\partial Z_w}{\partial T_h} = {}^{\circ}Z_{ws}\frac{\partial RH}{\partial T_h}$$

But RH $\approx$ 100-5($T_a - T_d$) (Lawrence 2005) Where T_a and T_d are temperatures of the air and dew point temperatures, with

$T_a = f(T_h)$. while assuming for simplicity that the relationship is linear , such that $\beta T_h = T_a$ where $\beta < 1$

$\frac{\partial RH}{\partial T_h} = -\beta 5$ since $\frac{\partial T_d}{\partial T_h} = 0$

$$\frac{\partial \delta}{\partial T_h} = -\beta\,{}^{\circ}Z_{ws}\,5 * \frac{A\,{}^{\circ}D_w}{{}^{\circ}m_w} \longrightarrow$$ Major Equation 3:

Recalling that :

$$\frac{\partial D}{\partial T_h} = -\left[{}^{G}/_{\delta^2 A^2} \times \left(\frac{A\partial\delta}{\partial T_h} + \frac{\delta\partial A}{\partial T_h}\right)\right] = 0 \text{ for optimum}$$

Simplifies to $\frac{A\partial\delta}{\partial T_h} = -\frac{\delta\partial A}{\partial T_h}$ $\longrightarrow$ Major Equation 4:

Therefore combining ‘Major Equations’, 1, 3 and 4

$$\frac{\beta\,{}^{\circ}Z_{ws}5A^2\,{}^{\circ}D_w}{{}^{\circ}m_w} = 4\delta\alpha\left[E_p T^3{}_p - E_h T^3{}_h\right]$$

However mass of moisture(${}^{\circ}m_w$) = density × distance × cross sectional area

$${}^{\circ}m_w = {}^{\circ}\rho_w\,\delta A$$

$$\frac{\beta\,{}^{\circ}Z_{ws}5A\,{}^{\circ}D_w}{{}^{\circ}\rho_w\,\delta} = 4\delta\alpha\left[E_p T^3{}_p - E_h T^3{}_h\right]$$

Therefore to optimise the drying operation

$$\frac{\delta^2}{A} = \frac{1.25\beta\,{}^{\circ}Z_{ws}\,{}^{\circ}D_w}{\alpha\,{}^{\circ}\rho_w\left[E_p T^3{}_p - E_h T^3{}_h\right]} \longrightarrow$$ Final Equation 5:

Therefore to optimise the drying operation for a porous body

A = Tδ^2, where T = constant of proportionality

VI. CONCLUSION

This report was expected to the drying operation of a porous body while investigating the influence of the surface area and the theoretical gap. This approach was greatly simplified to aid analysis via an incorporation of a series of assumptions , the most significant being the assumption of steady state operation. The investigation was therefore undertaken via the extensive consideration of the distinct aspects of the drying process:

- **Drying via radiation**

  Drying through radiation was considered via a comprehensive investigation of the Stefan-Boltzmann law, with respect to a porous body facilitating the development of relevant models for optimization.

- **Drying via convection**

Drying through convection was considered via a comprehensive investigation of expressions that incorporated mass transfer parameters, leading to the generation of models that reinforced the assertion that the drying operation involved simultaneous mass and heat transfer processes.

The critical significance of the drying process to industrial operations was also explored as well as the need for a more efficient drying process that will minimize energy losses subsequently thus also ensure that the need for sustainability is upheld.

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